Definition 6.10 (Quotient Set). *A quotient set of universe $\mathscr{X}$, with respect to equivalence relation* R*, is the set of all equivalence classes of $\mathscr{X}$ due to* R *and it is denoted by $\mathscr{Q} = \mathscr{X}/\mathsf{R}$.*

Definition 6.11 (Parition). *A partition, $\mathscr{P}$, is a quotient set of $\mathscr{X}$ according to a partition equivalence relation,* P*. Therefore,*

$$\mathscr{P} = \mathscr{X}/\mathsf{P} \tag{6.5}$$

where P *is generally designed to split the universal set into equivalence classes having some desired features.*

As we shall see later, an *equivalence relation* is quite similar to the concept of a *measure* which will be defined in Section 6.2. It will become more clear as we cover more equivalence concepts in this section and when we continue with the treatment of *measure theory* and the concept of a *measurable space*. In fact, we shall see that an *equivalence relation* may be viewed as a discrete measure in space $\mathscr{X}$, creating a *measure space*, $(\mathscr{X}, \mathfrak{X}, \mathsf{R})$. $\mathfrak{X}$ would then be a *Borel field* of $\mathscr{X}$ and R is the measure.

All the set theoretic definitions up to this point have made the assumption that objects either belong to a set or they do not. This, so called, *crisp logic*, responds to the question of equivalence in a binary fashion. Other types of sets have been developed in the past few decades which handle the concept of *uncertainty* in membership. This *uncertainty* may be viewed as the existence of objects in the boundary that a set shares with its complement, such that the membership of these objects into the set $\mathscr{A}$, and its complement $\mathscr{A}^{\complement} = \mathscr{X} \setminus \mathscr{A}$, is defined by *non-crisp logic* or in other words through an *uncertain membership*.

A generic two-class *crisp partition* may be denoted as follows, $\mathscr{P} = \{\mathscr{A}, \mathscr{X}\setminus\mathscr{A}\}$. In a *crisp* partitioning logic, we may define a binary function associated with set $\mathscr{A}$, called the *characteristic function* and denoted by $\Upsilon_{\mathscr{A}}(x)$. This function defines the membership of object x to set $\mathscr{A}$ and is defined as follows,

$$\Upsilon_{\mathscr{A}}(x) \triangleq \begin{cases} 1 \ \forall \ \{x : x \in \mathscr{A}\} \\ 0 \ \forall \ \{x : x \notin \mathscr{A}\} \end{cases} \tag{6.6}$$

As we shall see later, the *characteristic function* of a general set need not be binary for sets that allow *soft membership* such as *rough sets* and *fuzzy sets*. However, the definition does require that in a *universe* consisting of Γ disjoint sets denoted by $\mathscr{A}_\gamma, \gamma \in \{1, 2, \cdots, \Gamma\}$,

$$0 \le \Upsilon_{\mathscr{A}_\gamma}(x) \le 1 \ \forall \ \gamma \in \{1, 2, \cdots, \Gamma\} \tag{6.7}$$

and