$$e_n = h_n - \sum_{q=1}^{Q} a_q h_{n-q} \tag{5.60}$$

This is the error for each sample in the frame. Now, the problem is to find the parameters, a_q, such that we have minimum error in some sense. We have to decide on the metric that we would like to choose for the minimization. A popular choice is to minimize the sum of squares of errors over all the samples in the frame. This means that our minimization objective function will be,

$$\begin{aligned} E &= \sum_{n=0}^{N-1} E_n \\ &= \sum_{n=0}^{N-1} e_n^2 \\ &= \sum_{n=0}^{N-1} \left[h_n - \sum_{q=1}^{Q} a_q h_{n-q} \right]^2 \end{aligned} \tag{5.61}$$

where E_n is the square of the error for sample n and E is the sum of squares of errors over all N samples in the frame.

To minimize E, we can take its partial derivative with respect to the AR parameters, a_q, and set it equal to zero and then solve for the a_q,

$$\frac{\partial E}{\partial a_q} = 0 \tag{5.62}$$

The solution to problem 5.62 has been attacked from two different perspectives. Rabiner and Juang [60] use the signal samples in a similar fashion as we have discussed here, with a slight difference in handling the particular part of the solution to the difference equation. However, after taking the derivative, the same results are obtained in their approach and the one we have presented here.

Makhoul [46] has approached the problem from its *dual* perspective. Namely, he has defined the error in the spectral domain as,

$$\begin{aligned} E_s &= \frac{G^2}{2\pi} \int_{-\pi}^{\pi} \frac{\mathscr{P}_d^{\circ}(\omega)}{\hat{\mathscr{P}}_d^{\circ}(\omega)} d\omega \\ &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathscr{P}_d^{\circ}(\omega) \left| 1 - \sum_{q=1}^{Q} a_q e^{-i\omega q} \right|^2 d\omega \end{aligned} \tag{5.63}$$

where E_s is the error defined by [46] in the spectral domain, $\hat{\mathscr{P}}_d^{\circ}(\omega)$ is the *all-pole* estimate of the PSD given by Equation 5.55, and $\mathscr{P}_d^{\circ}(\omega)$ is the true PSD. Therefore, the minimization problem according to [46] becomes,

$$\frac{\partial E_s}{\partial a_q} = 0 \tag{5.64}$$